Outline of Solutions to the re-Exam in Financial Econometrics A: February 2013 (for fall term 2012)

Anders Rahbek, University of Copenhagen

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Question A:

Question A.1:Solution: Markov Chain. One should verify conditions (Gaussian transition density etc). Drift function $\delta(x) = 1 + x^2$, gives:

$$E(\delta(x_t)|x_{t-1} = x) = 1 + (\omega + \alpha|x|)^2 = 1 + \omega^2 + 2\omega\alpha|x| + \alpha^2 x^2$$

Hence $\alpha < 1$ (reasoning should be included).

Question A.2: Solution:

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \alpha}\bigg|_{\theta=\theta_0} = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left(1 - z_t^2\right) q_t \quad \text{where } q_t = \frac{2|x_{t-1}|}{1+2|x_{t-1}|}.$$

$$\xrightarrow{D} N(0, \omega_S).$$

This is a MGD (should be verified) and as $q_t^2 \leq 1$ the LLN for weakly mixing processes give the needed with $\omega_S = 2E\left(q_t^2\right)$.

Question A.3: Solution: LLN (weakly mixing) gives (q_t^2 is bounded by a constant), at $\theta = \theta_0$,

$$E\left(2_t^2 - 1\right) Eq_t^2 = Eq_t^2 = \omega_I = \frac{1}{2}\omega_S$$

Collecting terms (using Theorem III.2 - need to discuss this) we get (up to a constant) $\sqrt{T}\left(\hat{\theta} - \theta_0\right) \to N(0, v)$ where v is proportional to ω_S.

Question A.4: Solution:

Well-specified residuals (assuming also no-ARCH is not rejected based on the graph.

With $\delta_0 = 1$ can be accepted - and $\beta_0 = 0$ (one should explicitly compute t–statistics): the model is nested and not rejected.

Question B:

Question B.1: Solution: "Bubbles" modelled by switching between regimes with random walk behavior (maybe better if explosive?) and iid behavior. Usual comments re. p_{ii} close to 1 and model well-specified (Gaussian "residuals", no ARCH accepted). Clearly accepted a unit-root in one regime: The random walk regime has highest variance it seems. No std. errors reported so more cannot be said.

Question B.2: Solution:

$$f_\theta\left(y_t|y_{t-1}\right) = \frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-\frac{\left(y_t-\rho y_{t-1}\right)^2}{2\sigma_1^2}\right)$$

Question B.3: Solution: EM-algorithm - should be discussed in detail. In particular, (i) smoothed probabilities $p_t^*\left(\cdot\right)$, and (ii) how these are used in the recursive algorithm. Fig B.2 shows the smoothed probabilities $p_t^*\left(1\right)$ estimated- seems likely to spend much time in unit-root regime.

Question B.4: Solution:

$$\begin{aligned}
P\left(s_{T+2}=1|s_T=1\right) &= \frac{P(s_{T+2}=1,s_T=1)}{P(s_T=1)} = \frac{P(s_{T+2}=1,s_{T+1}=1,s_T=1)}{P(s_T=1)} + \frac{P(s_{T+2}=1,s_{T+1}=2,s_T=1)}{P(s_T=1)} \\
&= P\left(s_{T+2}=1|s_{T+1}=1\right)P\left(s_{T+1}=1|s_{T+1}=1\right) \\
&\quad +P\left(s_{T+2}=1|s_{T+1}=2\right)P\left(s_{T+1}=2|s_{T+1}=1\right) \\
&= p_{11}^2 + p_{12}p_{21} \\
\hat{p}_{11}^2 + \hat{p}_{12}\hat{p}_{21} &= 0.95^2 + 0.05 * 0.07 = 0.9
\end{aligned}$$

Hence very likely to enter regime 1.

Question B.5: Solution: The regime switching implies that $\alpha+\beta=1$ due to the misspecification. One couould elaborate here: Provide the usual discussion of GARCH models as "filtering" - here it seems a bad and very strange idea to suggest this in fact.